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ROYAL SIGNALS & RADAR ESTABLISHMENT

MONTE CARLO ANALYSIS OF
LATERAL COLLECTION DIODE ARRAYS

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AUTHORS: A P Davis and A M White

DATE: October 1987

SUMMARY

The determination of the thermally generated, diffusion limited leakage current density of lateral collection diode arrays is not possible analytically except for those with the simplest of geometries. It is shown here how the simulation of a spot-scan experiment, using a Monte Carlo analysis, can be used to determine the leakage current density of an array of any geometry. In particular, it is used to analyse a diode array of hexagonal symmetry as a function of array geometry and bulk diffusion length. The analysis shows that the leakage current of a diode may be substantially reduced by the presence of nearby reverse-biassed diodes.

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MONTE CARLO ANALYSIS OF LATERAL COLLECTION DIODE ARRAYS

A P Davis and A M White

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1   INTRODUCTION

Lateral collection photodiodes, utilising loophole fabrication technology, are an important aspect of current cadmium mercury telluride (CMT) detector technology. The diodes are fabricated in thin material and penetrate the full thickness so that minority carriers are collected from the surrounding (ie lateral) regions[1]. Arrays of such diodes may be employed to form devices such as bipolar transistors[2], proximity extraction diodes[3], and focal plane infrared detector arrays.

The magnitude of the thermally generated leakage current is of major importance in such devices. In one type of device, one or more reverse-biassed diodes are situated close to the main diode. The purpose of the additional diodes is to reduce the minority carrier concentration in the neighbourhood of the principal diode, and consequently also reduce the majority carrier concentration. The reduction in the electron concentration suppresses the Auger generation, and this reduces the leakage current density of the main diode. This process is referred to as proximity extraction. However, unless the array is of extremely simple (one-dimensional) geometry, it is not possible to determine the magnitude of the leakage current analytically and it must be determined by simulation.

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·The modelling of the leakage current by simulating carrier generation and diffusion from every point in the whole area of the device would be extremely time-consuming. However, it has been shown[4-7] that the leakage current density may be related to the local apparent value of the minority carrier diffusion length L' measured at the junction by a spot-scan experiment. This experiment involves the determination of the photocurrent i of the reverse-biassed diode as a function of the position x of a point source of optical excitation as it is scanned across the diode normal to the junction. To determine L', it is only necessary to determine the values of i and di/dx close to the junction. This value of L' then replaces the actual value of the diffusion length L in the formula relating leakage current density to diffusion length (see below), and gives an exact result for diffusion-limited diodes of arbitrary shape in arbitrarily non-uniform material. Thus, the simulation of a spot-scan may be used to determine the leakage current and requires considerably less computation ($\sim \times 10^{-4}$) than the simulation of the leakage current directly.

2   GENERAL THEORY

This section briefly reviews the analysis given in Refs 4-7. The apparent local diffusion length L' is defined by

$$L' = -i/(di/dx) = -1/d(\ln i)/dx \qquad (1)$$

Thus, a value for L' may be obtained by determining the collection efficiency η, which is proportional to i, at two points close to the junction. The thermally generated diffusion limited leakage current density J at this point on the junction can be obtained from this value of L' from

$$J = qu_0 D/L' \qquad (2)$$

where q is the carrier charge, u_0 is the thermal equilibrium minority carrier concentration, and D is the minority carrier diffusion constant.

The analysis on which this result depends assumes that the excitation in the spot-scan is uniform through the thickness of the diode. This will be approximately true as the diodes are typically only 2-3 absorption lengths thick and transverse diffusion is very rapid. In the examples considered here, the analysis is essentially two-dimensional, which requires the diode thickness to be much less than the diffusion length L. The analysis can be extended to three dimensions without modification. The analysis also depends on the assumption that the process is diffusion limited, which will only be true if the bias field in the device is small. In addition, the analysis ignores Auger processes, but this is acceptable as the device is designed to be Auger suppressed, ie the carrier concentrations are driven to low enough values that the Auger generation is quenched.

Extra diodes situated close to the diode under test may be reverse-biassed or short-circuited[7]. When they are reverse-biassed, they will collect some of the carriers generated during the spot-scan experiment on the first diode. The extra diodes may be connected to the first diode, and the total spot-scan collection current measured. A spot-scan across the first diode will give a value for L', and the leakage current density at a particular point on the junction. Successive spot-scans at different points along the length of the junction will then determine the total leakage current density in the presence of the external reverse-biassed diodes. Because the extra diodes make the total collection more efficient, the inclusion of their collection currents necessarily increases L', so that the presence of the extra diodes leads to a decrease in the leakage current, as opposed to an increase in the spot-scan current.

The effect of additional diodes which are short–circuited may be modelled in a similar way, by ignoring their collection currents, because short–circuited junctions are simply surfaces of infinite surface recombination velocity, which are covered by the general spot–scan theory outlined above.

3   MONTE CARLO ANALYSIS

Values for η can be determined by a Monte Carlo analysis whereby a large number of carriers are generated at a point P, at a perpendicular distance x from the junction and the number captured by the junction after random walks determined. Figure 1 shows the principle of this. At each step in the walk, a random jump length and a random direction for the jump are determined. The jump length s follows an exponential distribution $e^{-s/\lambda}$, where λ is the mean free path. In a two–dimensional situation, the direction of the jump is uniformly distributed over 2π radians. After a jump, the co-ordinates of the carrier may fall within the junction boundary, and the carrier is deemed to be captured. If it is not captured, then a random number is generated to determine whether it will recombine, using a probability for it living of

$$P = e^{-s\tau_s/\lambda\tau}$$

(3)

where τ is the minority carrier lifetime and τ_s is the scattering time, ie the time taken to travel a distance λ. The bulk diffusion length L is given by

$$L = \lambda\sqrt{\frac{\tau}{N\tau_s}}$$

(4)

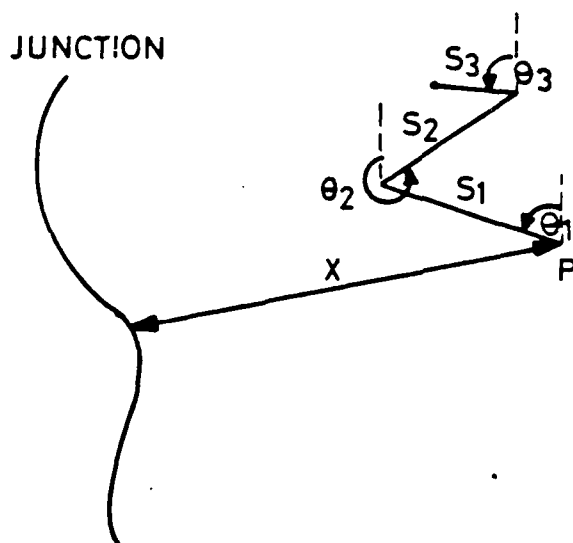N is the number of dimensions of the geometry, so in the cases to be considered here, N = 2.

Fig.1. PRINCIPAL OF MONTE CARLO ANALYSIS

The standard deviation of the calculated value of η is

$$\sigma_\eta = \sqrt{\frac{\eta(1-\eta)}{M}} \qquad (5)$$

where M is the number of carriers used in the calculation.

A difficulty arises due to the need to use a finite sized mean free path λ. Typically for CMT at 253K, λ is of the order of 0.04 μm for electrons and 0.004 μm for holes. Such small values would increase the computation time by a factor of up to 600 compared with the time for the values we have used, $\lambda \leq 0.1$ μm. It can be shown that for a finite sized λ (or τ_s/τ), the capture efficiency in the one-dimensional case, normally identified as $e^{-x/L}$, is strictly

$$\eta = h \, e^{(h-1)x/\lambda} \qquad (6)$$

where

$$h = 1 + \frac{\tau_s}{\tau} - \left\{ \frac{\tau_s}{\tau} \left[1 + \frac{\tau_s}{\tau} \right] \right\}^{\frac{1}{2}}$$

which reduces to $e^{-x/L}$ as $\tau_s/\tau \to 0$. Consequently, the otherwise expected point $\eta = 1$, $x = 0$ cannot be included in the data. For small τ_s/τ the collection efficiency extrapolates to 1 only at $x = -\lambda$.

The corresponding simple two-dimensional case has not been solved analytically, although by numerical checking it is found that $\eta = h$ exactly at $x = 0$ and extrapolates very close to 1 at $x = -\lambda$. For our more complex geometries, we find that the error introduced by taking $\eta = 1$ at $x = -\lambda$, although systematic, is much less than the random errors obtained by only taking the efficiencies from closely neighbouring spot points, which are subject to statistical fluctuations due to the use of a finite number of carriers. Since the plot of $\ln(\eta)$ against x is curved, it is valueless to take spot measurements too far away from the junction itself. The use of the point $\eta = 1$, $x = -\lambda$ has the additional benefit of reducing the total computing time required.

4   ANALYTICALLY SOLVABLE EXAMPLE

As an example which can be solved analytically as well as by using the Monte Carlo analysis, consider two infinite linear diodes distance d apart as shown in Figure 2. A line source of optical excitation, parallel to the junctions and of density G, is incident on the diode at $x = x_0$. (A line source rather than a point source is used to reduce the problem to one-dimension.) Ignoring Auger processes, and with u_L and u_R representing the minority carrier concentrations to the left and right of x_0 respectively, the continuity equations are

$$D \frac{d^2 u_L}{dx^2} = \frac{u_L}{\tau} \quad \text{and} \quad D \frac{d^2 u_L}{dx^2} = \frac{u_R}{\tau} \qquad (7)$$

where $L^2 = D\tau$. With both junctions reverse-biassed, the boundary conditions are $u_L(0) = u_R(d) = 0$ and $u_L(x_0) = u_R(x_0)$. Also at $x = x_0$,

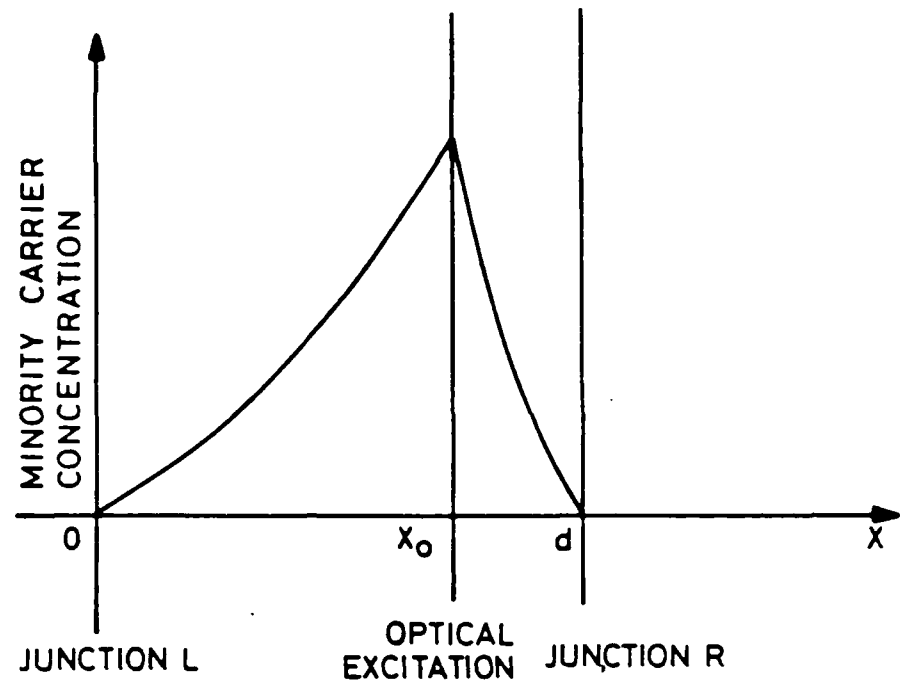$$G = D \left\{ \left[\frac{du_L}{dx} \right] - \left[\frac{du_R}{dx} \right] \right\} \qquad (8)$$

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Fig. 2. DOUBLE INFINITE LINEAR DIODE IN REVERSE BIAS

giving

$$u_L = \frac{LG}{D} \frac{\sinh([d-x_o]/L)}{\sinh(d/L)} \cdot \sinh(x/L)$$

$$u_R = \frac{LG}{D} \frac{\sinh(x_o/L)}{\sinh(d/L)} \cdot \sinh([d-x]/L)$$

(9)

The total collection current is equal to the sum of the minority carrier diffusion currents at $x = 0$ and $x = d$,

$$i = Dq \left\{ \left[\frac{du_L}{dx} \right]_{x=0} - \left[\frac{du_R}{dx} \right]_{x=d} \right\}$$

$$= qG \frac{(\sinh([d-x_o]/L + \sinh(x_o/L))}{\sinh(d/L)}$$

(10)

Differentiating i with respect to x_o gives

$$\frac{di}{dx_o} = \frac{qG}{L} \cdot \frac{(-\cosh([d-x_o]/L) + \cosh(x_o/L))}{\sinh(d/L)}$$

(11)

5

L', defined as $-i/(di/dx_0)$, is then given at $x_0 = 0$ by

$$L' = \frac{-\sinh(d/L)}{1 - \cosh(d/L)} \cdot L$$

$$= \frac{(1 + e^{-d/L})}{(1 - e^{-d/L})} \cdot L \tag{12}$$

To obtain the value of i for uniform generation, equation (10) may be integrated over x_0, putting G/dx_0 equal to u_0/τ for thermal generation. Alternatively, in the absence of optical excitation, the continuity equation for minority carriers is given by

$$D \frac{d^2 u}{dx^2} - \frac{u - u_0}{\tau} = 0 \tag{13}$$

With both diodes reverse-biassed, the boundary conditions are $u(0) = u(d) = 0$, and the solution to the above equation is

$$u = \frac{u_0}{(1 + e^{-d/L})} (1 - e^{-x/L} - e^{-(d-x)/L} + e^{-d/L}) \tag{14}$$

The leakage current density J in the diode at x = 0 is given by

$$J = Dq \left[\frac{du}{dx}\right]_{x=0} = Dqu_0/L' \tag{15}$$

with

$$L' = \frac{1 + e^{-d/L}}{1 - e^{-d/L}} \cdot L \tag{16}$$

Equation (16) is identical to equation (12). Thus, the leakage current density is inversely proportional to the apparent value of the minority carrier diffusion length L' at the junction, determined from a spot-scan.

In the limit of $d \rightarrow \infty$, $L' \rightarrow L$, a well-known result for a single infinite linear diode. In the opposite limit of $d << L$, $L' \rightarrow 2L^2/d$. This latter result may also be obtained directly from

$$J = qg_0 d/2 \tag{17}$$

where $g_0 = u_0/\tau$ is the thermal generation rate.

A Monte Carlo analysis was carried out for a double infinite linear diode with $d = 5$ μm and for a single infinite linear diode, both with a bulk diffusion length L of 20 μm. The aim of this calculation was to test the accuracy of the method with various values of λ and x. In order to minimise computing time, λ should be as long and x as small as possible. However, the two values cannot be chosen independently since a smaller value of x requires a smaller value of λ for accuracy. In addition, as mentioned above, x must be close enough to the junction that the value determined for L' is that at the junction, ie the plot of $\ln(\eta)$ against x must remain linear over the values of x used.

The results of the analysis are shown in Table 1, and may be compared with the analytically determined answers of 160.8 μm and 20.0 μm for the double and single diodes respectively. Using values for $\lambda/L \leq 5 \times 10^{-3}$ and $x/\lambda \leq 5$, the errors in the values obtained for L' are within the errors predicted by equation (6). This indicates that these values for λ and x are small enough not to introduce extra errors.

M	λ μm	x μm	η	L' υm
		double diode d = 5 μm		
20000	0.02	0.10	0.9993	160
10000	0.10	0.25	0.9978	159
10000	0.10	0.30	0.9978	182
10000	0.10	0.35	0.9967	136
10000	0.10	0.40	0.9967	151
10000	0.10	0.45	0.9966	161
10000	0.10	0.50	0.9961	154
		single diode		
10000	0.05	0.50	0.9730	20.1
20000	0.05	1.00	0.9489	20.0
10000	0.10	1.00	0.9456	19.7
10000	0.10	1.50	0.9235	20.1
10000	0.10	2.00	0.8980	19.5

TABLE 1.   Results of analysis of single and double infinite linear diode

5   HEXAGONAL SYMMETRY PROXIMITY EXTRACTION DEVICE

Figure 3 shows a lateral collection diode array of hexagonal symmetry in which the central diode C is surrounded by an inner ring R1 of six similar diodes and an outer ring R2 of twelve further diodes[2,3]. The central diode is reverse-biassed, and the two outer rings may be operated in three different modes:

a.   R1 and R2 reverse-biassed

b.   R1 reverse-biassed and R2 short-circuited
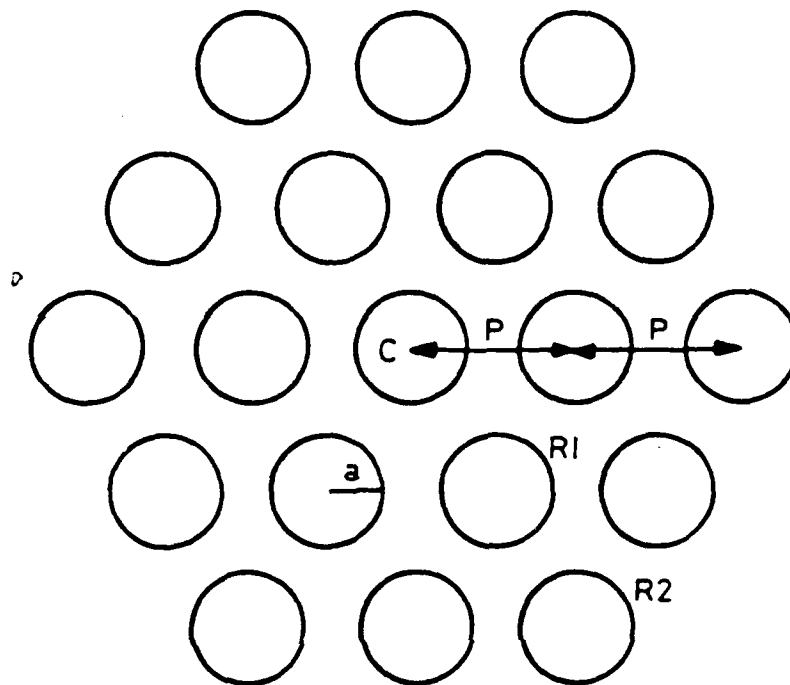
c.   R1 and R2 short-circuited

Fig. 3. HEXAGONAL SYMMETRY DIODE ARRAY

Because of the results quoted in the theory section, all of these modes can be examined simultaneously in the same random walk experiment, simply by tabulating separately the numbers of carriers captured in each diode. Modes a or b apply when the array is being operated as a proximity extraction device. In this case, the output from R1 (mode a) or R1 and R2 (mode b) is added to that from C. The leakage current densities for the three modes will be denoted by J_a, J_b and J_c.

An analysis was carried out on this array using various values for the pitch P and diffusion length L, and a diode diameter a of 10 μm. Advantage was taken of the hexagonal symmetry to limit the bounds of the paths taken to a 30° "slice", as shown in Figure 4.
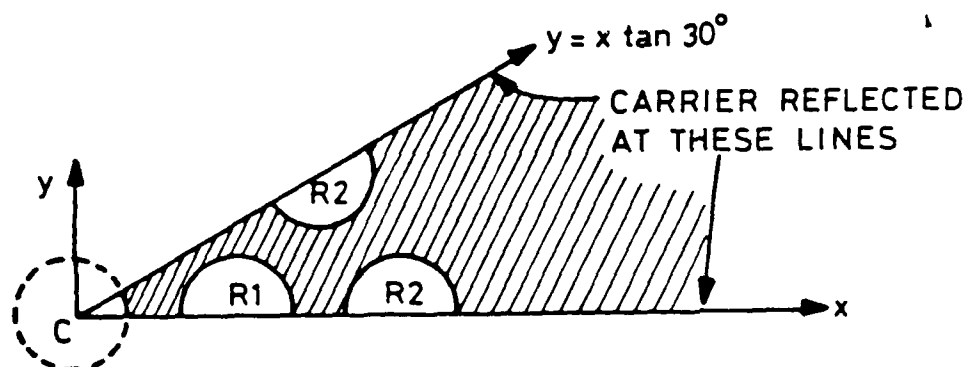

Fig. 4. LIMITS OF RANDOM WALKS

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When P is small in comparison to a, then the angular position of the spot–scan effects the result and it is necessary to determine η at several angles and average the current density over the length of the junction. When $P >> a$, the angular position of the spot–scan is not important and it is only necessary to determine η at one angle.

The resulting values of L' are shown in Table 2. Also shown are the values of L'_0 for an isolated diode, which may be determined analytically[5]. From these values, the ratio of the leakage current density in each mode to that of an isolated diode, J_x/J_0 (where x = a, b or c), may be determined. J_x/J_0 is plotted on a logarithmic scale in Figure 5 as a function of P. As P approaches the diameter of the diodes, the outer ring R2 becomes ineffective since there is very little space between the diodes of R1. Modes a and b therefore have similar, small leakage current densities, and because R1 has a high probability of capturing carriers, mode c has a high leakage current density. As P increases, J_a and J_b increase and begin to differ as R2 becomes more effective. J_c falls with increasing P. Eventually, as P increases well beyond the diffusion length L, the outer diodes become ineffective and the leakage current densities of all three modes tend to that of an isolated diode. This convergence is shown most clearly in the plot for $L = 15 \ \mu m$, which extends to higher values of P. Figure 6 shows an enlargement of the plots for J_a/J_0 and J_b/J_0 on a linear scale. It is clear that large reductions in leakage current can be achieved by the presence of nearby reverse–biassed diodes, although for the geometries considered here the outer ring R2 has little effect.

The ratios J_c/J_a and J_c/J_b represent the improvement in the device in its "on" (a and b) modes relative to its "off" (c) mode, and are plotted in Figure 7. The ratio of ~ 17 for $P = 15 \ \mu m$ and $L = 15 \ \mu m$ is in good agreement with an experimental result obtained with a non–Auger limited diode[8].

| L μm	L'_o μm	P μm	L'_a μm	L'_b μm	L'_c μm	J_a/J_o	J_b/J_o	J_c/J_o
15	7.07	11	248	248	1.26	.0285	.0285	5.60
		12.5	143	141	2.39	.0496	.0503	2.96
		15	66.4	62.6	3.82	.107	.113	1.85
		17.5	46.2	43.2	5.05	.153	.164	1.40
		20	30.6	28.4	5.82	.231	.249	1.22
		25	19.0	17.7	6.29	.372	.400	1.12
		30	13.7	13.0	6.87	.514	.544	1.03
		40	10.1	10.0	7.35	.699	.706	0.96
		50	8.60	8.56	7.53	.822	.826	0.94
		60	7.70	7.67	7.34	.912	.921	0.96
20	8.23	11	327	327	1.28	.0252	.0252	6.43
		12.5	182	182	2.37	.0453	.0453	3.47
		15	125	114	3.93	.0658	.0724	2.09
		17.5	74.9	65.2	5.10	.110	.126	1.61
		20	52.3	45.9	5.98	.157	.179	1.38
25	9.17	11	1800	1800	1.26	.00510	.00510	7.29
		12.5	428	428	2.37	.0214	.0214	3.86
		15	183	157	3.99	.0500	.0585	2.30
		17.5	117	91.5	5.10	.0782	.100	1.80
		20	82.8	65.2	5.97	.111	.141	1.54
30	9.97	11	3600	3600	1.26	.00277	.00277	7.92
		12.5	621	621	2.39	.0160	.0160	4.18
		15	290	232	4.03	.0344	.0430	2.47
		17.5	163	120	5.14	.0613	.0827	1.94
		20	124	91.5	6.11	.0805	.109	1.63

TABLE 2.   Results of Analysis of Hexagonal Diode Array
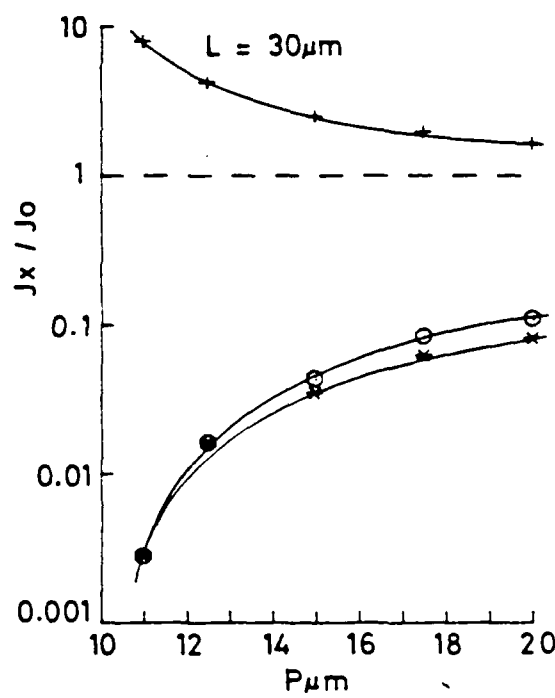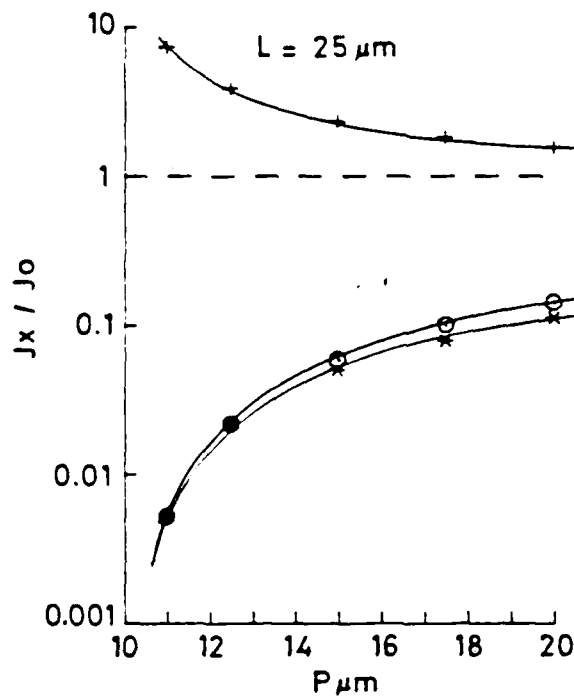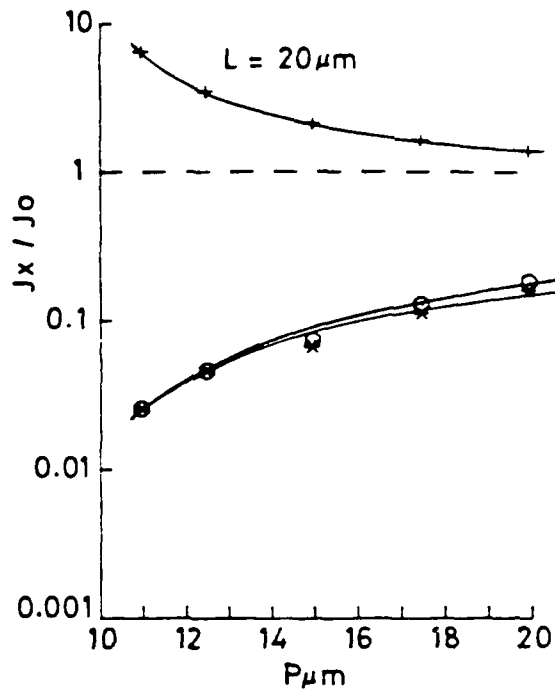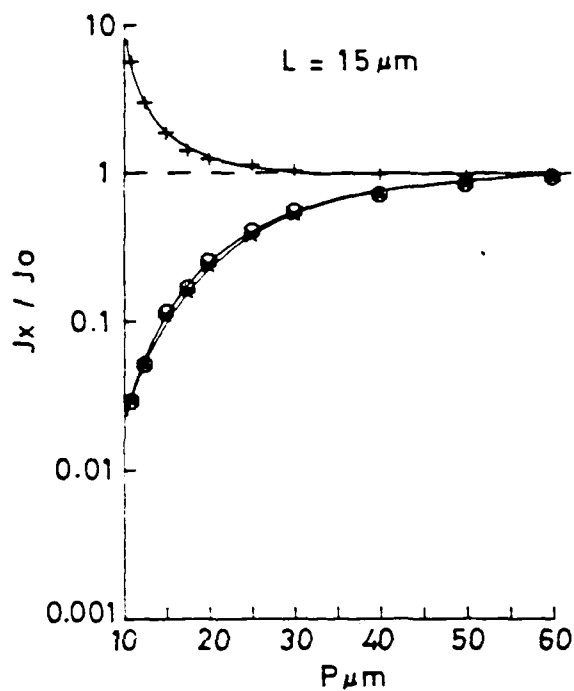
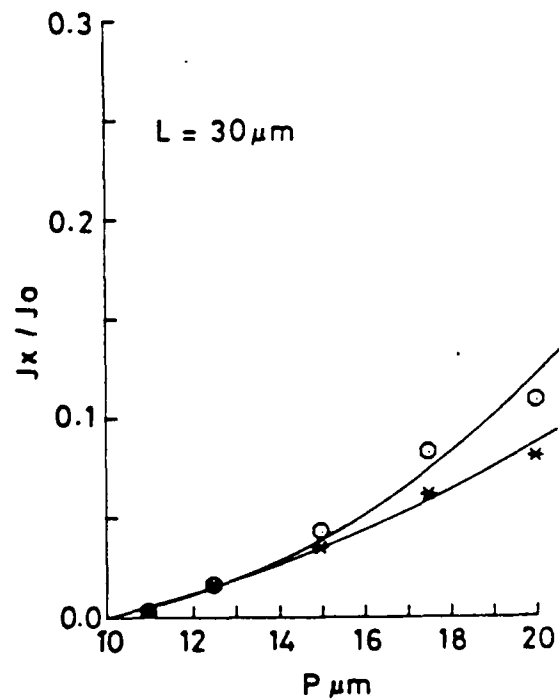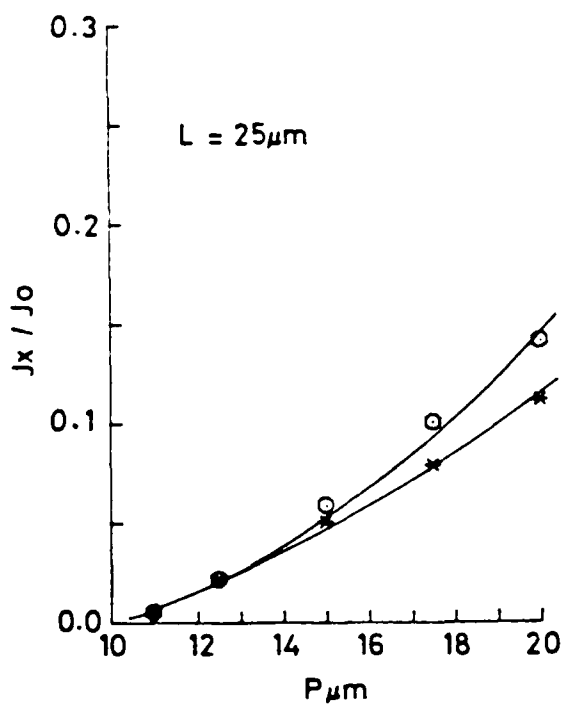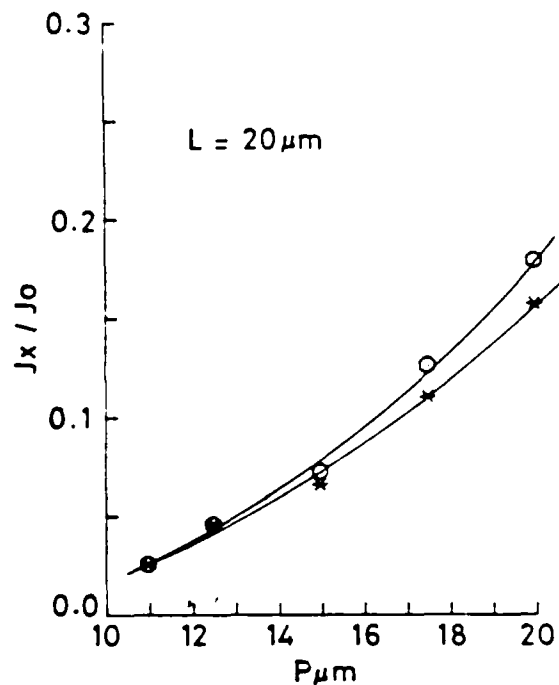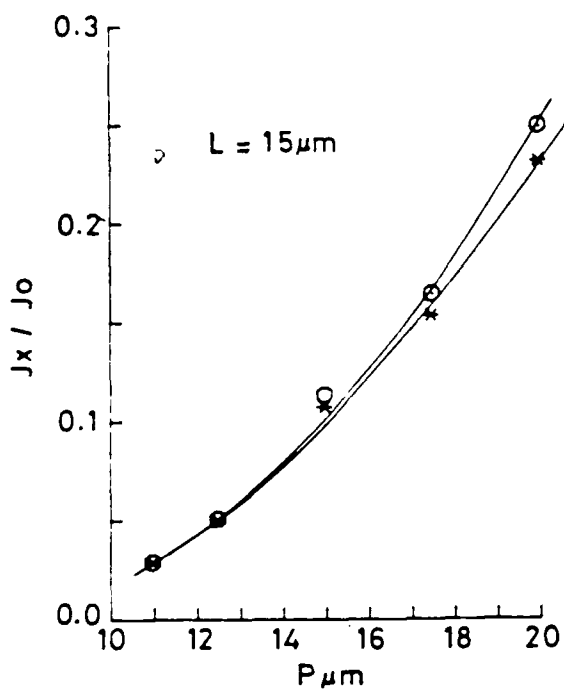FIG. 5 J_a/J_0 (∗) , J_b/J_0 (o) AND J_c/J_0 (+) AS FUNCTIONS OF P AND L

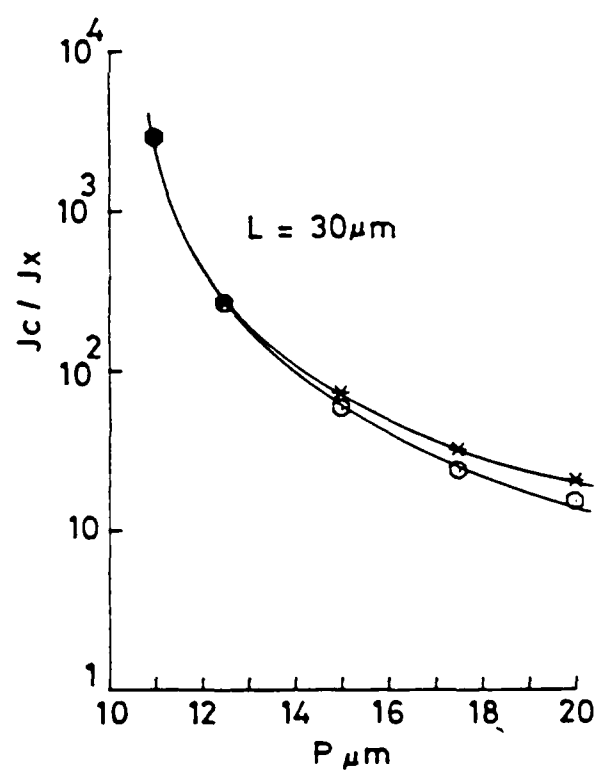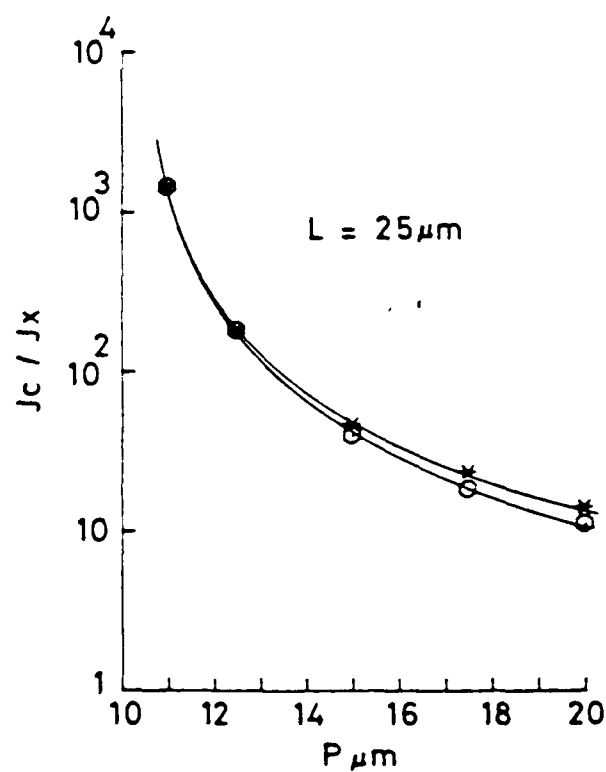FIG.6 ENLARGED LINEAR PLOTS OF J_a/J_o (∗) AND J_b/J_o (o) AS FUNCTIONS OF P AND L
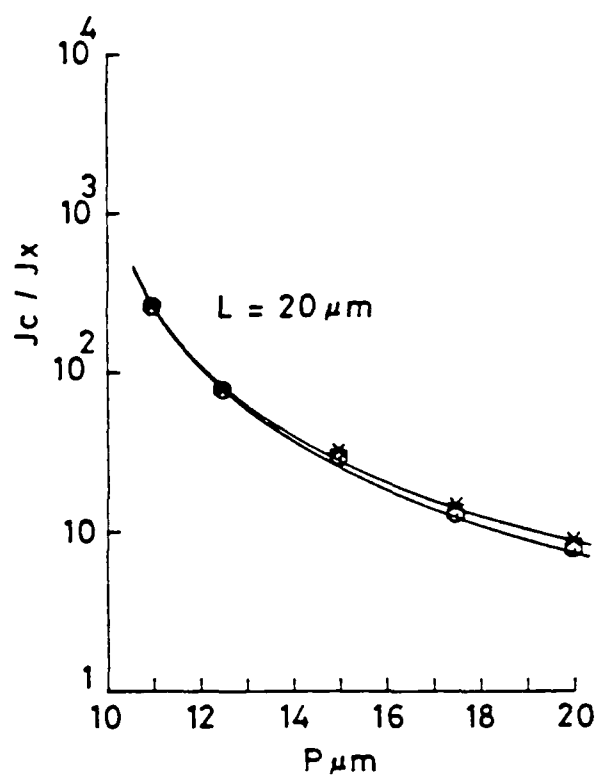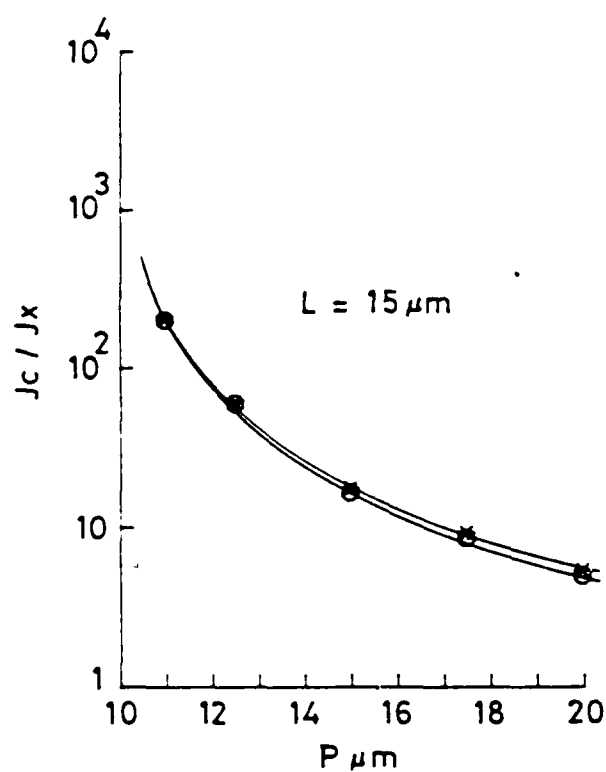
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FIG.7 J_c/J_a (*) AND J_c/J_b (o) AS FUNCTIONS OF P AND L

6 CONCLUSION

A method for determining the thermally generated, diffusion limited leakage current of an array of non-Auger limited lateral collection photodiodes has been demonstrated. The analysis uses a spot-scan simulation which is economical in computing time, and may be used for arrays of any complex geometry.

The analysis of a hexagonal symmetry diode array has shown that the leakage current of a diode may be substantially reduced, (typically by factors greater than 10), by the presence of nearby reverse-biassed diodes.

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Abstract

The determination of the thermally generated, diffusion limited leakage current
density of lateral collection diode arrays is not possible analytically except
for those with the simplest of geometries. It is shown here how the simulation
of a spot-scan experiment, using a Monte Carlo analysis, can be used to determine
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